MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2015/2016**

## BSA1024 – STATISTICS

( All sections / Groups )

1 MARCH 2016
2.30 p.m. - 4.30 p.m.
(2 Hours)

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**INSTRUCTIONS TO STUDENTS**

1. This question paper consists of **THIRTEEN (13)** printed pages with:
   **Section A:** Ten (10) multiple choice questions (20%)
   **Section B:** Three (3) structured questions (80%)

2. Answer **ALL** questions.

3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.

4. Students are allowed to use non-programmable scientific calculators with no restrictions.

## SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

**There are TEN (10) questions in this section. Answer ALL questions on the multiple choice answer sheet.**

1. The science of collecting, organizing, presenting, analyzing and interpreting data to assist in making more effective decisions is called ________.

   A. Statistic
   B. Parameter
   C. Population
   D. Statistics

2. When a variable studied can be reported numerically, the variable is called a ____

   A. Quantitative variable.
   B. Qualitative variable.
   C. Ordinal variable.
   D. Nominal variable.

3. Which of the following is a quantitative variable?

   A. Second quartile A person's gender
   B. The distance from one city to another (in km)
   C. A person's educational background
   D. Whether or not a person owns a credit card

4. Why does the standard deviation formula have a square root as part of it?

   A. To make it add up to the mean
   B. To reverse the effect of squaring the deviations
   C. To provide a smaller value of measure for variation
   D. To produce more accurate measurement of data deviation from the mean

5. The attendance of 4 cinema halls on a given day was 200, 500, 300 and 1000 people. Find the relative dispersion for the data.

   A. 30.82
   B. 71.18
   C. 0.7118
   D. 0.3082

**Continued...**

6. A manufacturer produces rolls of wallpaper. A flaw occurs when the patterns is not consistent. A 20 meter sample from each role is inspected. It was believed that the number of flaws per sample follows a Poisson distribution with a mean of one flaw per 20 meter sample. What is the probability that at least two flaws will appear in a 20 meter sample?

A. 0.7358
B. 0.2642
C. 0.3246
D. 0.4568

7. Regarding the hypothesis testing of the difference between two groups when comparing proportion, ________ is the criteria selecting either a t-test or a Z-test.

A. Value of the population standard deviation
B. Degree of freedom
C. Critical value
D. None of the above

8. Lumber companies need to estimate the amount of lumber that they can harvest in a tract of land to determine whether the effort will be profitable. To do so, they must estimate the mean diameter of the trees. It has been decided the estimation parameter is within 1 inch with 99% confidence. A forester familiar with the territory guesses that the diameters of the tree are normally distributed with a standard deviation of 6 inches. How large a sample should be taken?

A. 239
B. 390
C. 195
D. 235

9. In a past General Social Survey, a random sample of men and women answered the question "Are you a member of any sports clubs?". Based on the sample data, 95% confidence intervals for the population proportion who would answer "yes" are 0.13 to 0.19 for women and 0.247 to 0.33 for men. Based on these results, you can reasonably conclude that

A. At least 25% of American men and American women belong to sports clubs.
B. At least 16% of American women belong to sports clubs.
C. There is a difference between the proportions of American men and American women who belong to sport clubs.
D. There is no conclusive evidence of a gender difference in the proportion belonging to sports clubs.

**Continued...**

10. The coefficient of correlation of linear regression is computed to be -0.95 means that

A. The relationship between two variables is weak.
B. The relationship between two variables is strong with positive direction.
C. The relationship between two variables is strong with negative direction.
D. Correlation coefficient cannot have this value.

**Continued...**

## SECTION B: STRUCTURED QUESTIONS (80 Marks)

**There are THREE questions in this section. Candidates MUST answer ALL THREE questions.**

### Question 1 (20 Marks)

The probability distribution for the number of job applications processed at a small agency during a typical week as given in table below:

| Number of Job Applications Processed | Probability |
|---|---|
| 5 | 0.05 |
| 6 | 0.10 |
| 7 | 0.15 |
| 8 | 0.20 |
| 9 | 0.20 |
| 10 | 0.15 |
| 11 | 0.10 |
| 12 | 0.05 |

a) Find the expected number of job applications processed per week? [4 marks]

b) Assuming that it takes agency's administrative assistant 2 hours to process a submitted job application, on average how many hours on a typical week will the administrative assistant spend processing incoming job application? [4 marks]

c) Find an interval with the property that the administrative assistant can be approximately 99% sure that the total amount of time (in hour) he spends each week processing incoming job applications will be in this interval. Assume that the sampling has been done for 35 weeks. [12 marks]

**Continued....**

**Question 2 (30 Marks)**

a) A customer service supervisor regularly conducts a survey of customer satisfaction. The result of the latest survey indicates that 8% of customers were not satisfied with the service they received at their last visit to the store. Out of the unsatisfied customers, only 22% return to the store within a year. From the group of satisfied customers, 64% return within the year.

i. Draw a tree diagram and its probabilities for the above scenario of customer satisfaction. [6 marks]

ii. A customer has entered the store. In response to your question, he informs you that it is less than 1 year since his last visit to the store. What is the probability that he was satisfied with the service he received?

[5 marks]

b) Warren Dinner has invested in nine different investments. The returns on the different investment are probabilistically independent, and each return follows a normal distribution with mean \$500 and standard deviation \$100.

i. What is the probability that Warren's return for first investment is more than \$550? [4 marks]

ii. What is the probability that Warren's total return is between \$4000 and \$5200. Assume that mean is $\mu = \sum_{i=1}^{9} \mu_i$ and the variance is $\sigma^2 = \sum_{i=1}^{9} \sigma^2{}_i$. [5 marks]

c) A diet doctor claims that the average Malaysia men are more than 20 pounds overweight. To test his claim, a random sample of 20 Malaysia men was weighed, and the difference between their actual weight and their ideal weight was calculated. The data are listed below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 16 | 23 | 18 | 41 | 22 | 18 | 23 | 19 | 22 | 15 |
| 18 | 35 | 16 | 15 | 17 | 19 | 23 | 15 | 16 | 26 |

Do these data allow us to infer at the 5% significance level that the doctor's claim is true? [10 marks]

**Continued...**

## Question 3 (30 Marks)

a) Students who apply to MBA programs must have the Graduate Management Admission Test (GMAT) score. The GMAT score is used as one of the indicators of how well a student is likely to perform in the MBA programs. To judge how well the GMAT score predicts MBA performance (grade point average values from 0 to 12), a sample of 12 graduates are taken. The data is listed as below. The regression result for the sample to examine the relationship between variables is shown as below.

| GMAT | GPA |
|---|---|
| 599 | 9.6 |
| 689 | 8.8 |
| 584 | 7.4 |
| 631 | 10 |
| 594 | 7.8 |
| 643 | 9.2 |
| 656 | 9.6 |
| 594 | 8.4 |
| 710 | 11.2 |
| 611 | 7.6 |
| 593 | 8.8 |
| 683 | 8 |

SUMMARY OUTPUT

| *Regression Statistics* | |
|---|---|
| Multiple R | 0.536483 |
| R Square | 0.287814 |
| Adjusted R Square | 0.216595 |
| Standard Error | 0.990892 |
| Observations | 12 |

ANOVA

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 1 | 3.967991 | 3.967991 | 4.041269 | 0.032139679 |
| Residual | 10 | 9.818675 | 0.981868 | | |
| Total | 11 | 13.78667 | | | |

| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* |
|---|---|---|---|---|
| Intercept | 0.149633 | 4.34563 | 0.034433 | 0.97321 |
| GMAT | 0.013787 | 0.006858 | 2.010291 | 0.07214 |

**Continued...**

i. What is the dependent variable and independent variable for the regression model above? [2 marks]

ii. State the null and alternative hypotheses for the regression model above. [2 marks]

iii. State the estimated linear regression equation. [2 marks]

iv. State and interpret the regression coefficients. [4 marks]

v. Compute the coefficient of the variability in MBA grade point can be accounted for by the GMAT score achieved by the student? How much is the unexplained variability? [3 marks]

vi. Does the independent variable provide a significant contribution to the model? Perform the appropriate statistical inference at 5% significance level. Discuss the result. [4 marks]

vii. Predict the average of MBA grade point for two students who received 670 hours and 725 score points respectively. Are both predictions reliable? [6 marks]

b) Prices for selected foods for 1999 and 2005 are given in the following table.

| Item | 1999 | | 2005 | |
|---|---|---|---|---|
| | Price ($) | Quantity | Price ($) | Quantity |
| Cabbage (500 g) | 0.60 | 2000 | 0.90 | 1500 |
| Carrots (bunch) | 0.49 | 200 | 0.69 | 200 |
| Peas (kg) | 1.99 | 400 | 2.99 | 500 |
| Endive (bunch) | 0.89 | 100 | 1.29 | 200 |

i. Compute Laspeyres' and Paasche price index for 2005 using 1999 as the base year. [5 marks]

ii. Determine Fisher's ideal index using the values computed in (i). [2 marks]

**End of Page**

## A. DESCRIPTIVE STATISTICS

Mean ($\bar{x}$) = $\dfrac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation ($s$) = $\sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation ($CV$) = $\dfrac{\sigma}{\overline{X}} \times 100$

Pearson's Coefficient of Skewness ($S_k$) = $\dfrac{3(\overline{X} - Median)}{s}$

## B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B) if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

**Poisson Probability Distribution**

If $X$ follows a Poisson Distribution , $P(\lambda)$ where $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

**Binomial Probability Distribution**

If $X$ follows a Binomial Distribution B($n$, $p$) where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

**Normal Distribution**

If $X$ follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \dfrac{X - \mu}{\sigma}$

## C. EXPECTATION AND VARIANCE OPERATORS

$E(X) = \sum[X \bullet P(X)]$

$VAR(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \sum[X^2 \bullet P(X)]$

If $E(X) = \mu$ then $E(cX) = c\,\mu$, $E(X_1 + X_2) = E(X_1) + E(X_2)$

If $VAR(X) = \sigma^2$ then $VAR(cX) = c^2\sigma^2$, $VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,COV(X_1, X_2)$

where $COV(X_1, X_2) = E(X_1X_2) - [E(X_1)\,E(X_2)]$

## D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

(100 – α) % Confidence Interval for Population Mean (σ Known) = $\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma / \sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Mean (σ Unknown) = $\mu = \overline{X} \pm t_{\alpha/2,n-1}\left(s / \sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Proportion $= \hat{p} \pm Z_{\alpha/2}\sigma_{p^\wedge}$ Where $\sigma_{\hat{p}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean $= n \geq \left[\dfrac{(Z_{\alpha/2})\sigma}{E}\right]^2$

Sample Size Determination for Population Proportion $= n \geq \dfrac{(Z_{\alpha/2})^2\ \hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

## E. HYPOTHESIS TESTING

**One Sample Mean Test**

| **Standard Deviation (σ) Known** | **Standard Deviation (σ) Not Known** |
|---|---|
| $z = \dfrac{x - \mu}{\sigma / \sqrt{n}}$ | $t = \dfrac{x - \mu}{s / \sqrt{n}}$ |

**One Sample Proportion Test**

$z = \dfrac{\hat{p} - p}{\sigma_p}$ where $\sigma_p = \sqrt{\dfrac{p(1-p)}{n}}$

***Two Sample Mean Test***

| **Standard Deviation (σ) Known** | **Standard Deviation (σ) Not Known** |
|---|---|
| $z = \dfrac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$ | $t = \dfrac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$ |

**Two Sample Proportion Test**

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $p = \dfrac{X_1 + X_2}{n_1 + n_2}$

*where $X_1$ and $X_2$ are the number of successes from each population*

## F. REGRESSION ANALYSIS

**Simple Linear Regression**

**Population Model:** $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

**Sample Model:** $\hat{y} = b_0 + b_1 x_1 + e$

**Correlation Coefficient**

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left(\frac{(\sum X)^2}{n}\right)\right]\left[\sum Y^2 - \left(\frac{(\sum Y)^2}{n}\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

**ANOVA Table for Regression**

| Source | Degrees of Freedom | Sum of Squares | Mean Squares |
|---|---|---|---|
| Regression | $1$ | SSR | MSR = SSR/1 |
| Error/Residual | $n-2$ | SSE | MSE = SSE/$(n-2)$ |
| Total | $n-1$ | SST | |

**Test Statistic for Significance of the Predictor Variable**

$t_i = \dfrac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

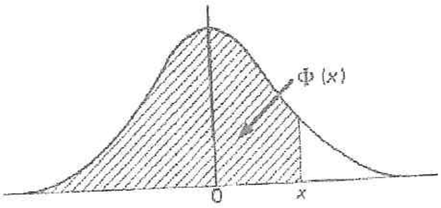
*Where p= number of predictor*

## G. INDEX NUMBERS

| | |
|---|---|
| **Simple Price Index** $P = \frac{p_t}{p_0} \times 100$ | **Laspeyres Quantity Index** $P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$ |
| **Aggregate Price Index** $P = \frac{\sum p_t}{\sum p_0}(100)$ | **Paasche Quantity Index** $P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$ |
| **Laspeyres Price Index** $P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$ | **Fisher's Ideal Price Index** $\sqrt{(Laspeyres\ Price\ Index)(Paa\ sche\ Price\ Index)}$ |
| **Paasche Price Index** $P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$ | **Value Index** $V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$ |

# TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to $x$. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.




| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0·00 | 0·5000 | 0·40 | 0·6554 | 0·80 | 0·7881 | 1·20 | 0·8849 | 1·60 | 0·9452 | 2·00 | 0·97725 |
| ·01 | ·5040 | ·41 | ·6591 | ·81 | ·7910 | ·21 | ·8869 | ·61 | ·9463 | ·01 | ·97778 |
| ·02 | ·5080 | ·42 | ·6628 | ·82 | ·7939 | ·22 | ·8888 | ·62 | ·9474 | ·02 | ·97831 |
| ·03 | ·5120 | ·43 | ·6664 | ·83 | ·7967 | ·23 | ·8907 | ·63 | ·9484 | ·03 | ·97882 |
| ·04 | ·5160 | ·44 | ·6700 | ·84 | ·7995 | ·24 | ·8925 | ·64 | ·9495 | ·04 | ·97932 |
| 0·05 | 0·5199 | 0·45 | 0·6736 | 0·85 | 0·8023 | 1·25 | 0·8944 | 1·65 | 0·9505 | 2·05 | 0·97982 |
| ·06 | ·5239 | ·46 | ·6772 | ·86 | ·8051 | ·26 | ·8962 | ·66 | ·9515 | ·06 | ·98030 |
| ·07 | ·5279 | ·47 | ·6808 | ·87 | ·8078 | ·27 | ·8980 | ·67 | ·9525 | ·07 | ·98077 |
| ·08 | ·5319 | ·48 | ·6844 | ·88 | ·8106 | ·28 | ·8997 | ·68 | ·9535 | ·08 | ·98124 |
| ·09 | ·5359 | ·49 | ·6879 | ·89 | ·8133 | ·29 | ·9015 | ·69 | ·9545 | ·09 | ·98169 |
| 0·10 | 0·5398 | 0·50 | 0·6915 | 0·90 | 0·8159 | 1·30 | 0·9032 | 1·70 | 0·9554 | 2·10 | 0·98214 |
| ·11 | ·5438 | ·51 | ·6950 | ·91 | ·8186 | ·31 | ·9049 | ·71 | ·9564 | ·11 | ·98257 |
| ·12 | ·5478 | ·52 | ·6985 | ·92 | ·8212 | ·32 | ·9066 | ·72 | ·9573 | ·12 | ·98300 |
| ·13 | ·5517 | ·53 | ·7019 | ·93 | ·8238 | ·33 | ·9082 | ·73 | ·9582 | ·13 | ·98341 |
| ·14 | ·5557 | ·54 | ·7054 | ·94 | ·8264 | ·34 | ·9099 | ·74 | ·9591 | ·14 | ·98382 |
| 0·15 | 0·5596 | 0·55 | 0·7088 | 0·95 | 0·8289 | 1·35 | 0·9115 | 1·75 | 0·9599 | 2·15 | 0·98422 |
| ·16 | ·5636 | ·56 | ·7123 | ·96 | ·8315 | ·36 | ·9131 | ·76 | ·9608 | ·16 | ·98461 |
| ·17 | ·5675 | ·57 | ·7157 | ·97 | ·8340 | ·37 | ·9147 | ·77 | ·9616 | ·17 | ·98500 |
| ·18 | ·5714 | ·58 | ·7190 | ·98 | ·8365 | ·38 | ·9162 | ·78 | ·9625 | ·18 | ·98537 |
| ·19 | ·5753 | ·59 | ·7224 | ·99 | ·8389 | ·39 | ·9177 | ·79 | ·9633 | ·19 | ·98574 |
| 0·20 | 0·5793 | 0·60 | 0·7257 | 1·00 | 0·8413 | 1·40 | 0·9192 | 1·80 | 0·9641 | 2·20 | 0·98610 |
| ·21 | ·5832 | ·61 | ·7291 | ·01 | ·8438 | ·41 | ·9207 | ·81 | ·9649 | ·21 | ·98645 |
| ·22 | ·5871 | ·62 | ·7324 | ·02 | ·8461 | ·42 | ·9222 | ·82 | ·9656 | ·22 | ·98679 |
| ·23 | ·5910 | ·63 | ·7357 | ·03 | ·8485 | ·43 | ·9236 | ·83 | ·9664 | ·23 | ·98713 |
| ·24 | ·5948 | ·64 | ·7389 | ·04 | ·8508 | ·44 | ·9251 | ·84 | ·9671 | ·24 | ·98745 |
| 0·25 | 0·5987 | 0·65 | 0·7422 | 1·05 | 0·8531 | 1·45 | 0·9265 | 1·85 | 0·9678 | 2·25 | 0·98778 |
| ·26 | ·6026 | ·66 | ·7454 | ·06 | ·8554 | ·46 | ·9279 | ·86 | ·9686 | ·26 | ·98809 |
| ·27 | ·6064 | ·67 | ·7486 | ·07 | ·8577 | ·47 | ·9292 | ·87 | ·9693 | ·27 | ·98840 |
| ·28 | ·6103 | ·68 | ·7517 | ·08 | ·8599 | ·48 | ·9306 | ·88 | ·9699 | ·28 | ·98870 |
| ·29 | ·6141 | ·69 | ·7549 | ·09 | ·8621 | ·49 | ·9319 | ·89 | ·9706 | ·29 | ·98899 |
| 0·30 | 0·6179 | 0·70 | 0·7580 | 1·10 | 0·8643 | 1·50 | 0·9332 | 1·90 | 0·9713 | 2·30 | 0·98928 |
| ·31 | ·6217 | ·71 | ·7611 | ·11 | ·8665 | ·51 | ·9345 | ·91 | ·9719 | ·31 | ·98956 |
| ·32 | ·6255 | ·72 | ·7642 | ·12 | ·8686 | ·52 | ·9357 | ·92 | ·9726 | ·32 | ·98983 |
| ·33 | ·6293 | ·73 | ·7673 | ·13 | ·8708 | ·53 | ·9370 | ·93 | ·9732 | ·33 | ·99010 |
| ·34 | ·6331 | ·74 | ·7704 | ·14 | ·8729 | ·54 | ·9382 | ·94 | ·9738 | ·34 | ·99036 |
| 0·35 | 0·6368 | 0·75 | 0·7734 | 1·15 | 0·8749 | 1·55 | 0·9394 | 1·95 | 0·9744 | 2·35 | 0·99061 |
| ·36 | ·6406 | ·76 | ·7764 | ·16 | ·8770 | ·56 | ·9406 | ·96 | ·9750 | ·36 | ·99086 |
| ·37 | ·6443 | ·77 | ·7794 | ·17 | ·8790 | ·57 | ·9418 | ·97 | ·9756 | ·37 | ·99111 |
| ·38 | ·6480 | ·78 | ·7823 | ·18 | ·8810 | ·58 | ·9429 | ·98 | ·9761 | ·38 | ·99134 |
| ·39 | ·6517 | ·79 | ·7852 | ·19 | ·8830 | ·59 | ·9441 | ·99 | ·9767 | ·39 | ·99158 |
| 0·40 | 0·6554 | 0·80 | 0·7881 | 1·20 | 0·8849 | 1·60 | 0·9452 | 2·00 | 0·9772 | 2·40 | 0·99180 |

# TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2·40 | 0·99180 | 2·55 | 0·99461 | 2·70 | 0·99653 | 2·85 | 0·99781 | 3·00 | 0·99865 | 3·15 | 0·99918 |
| ·41 | ·99202 | ·56 | ·99477 | ·71 | ·99664 | ·86 | ·99788 | ·01 | ·99869 | ·16 | ·99921 |
| ·42 | ·99224 | ·57 | ·99492 | ·72 | ·99674 | ·87 | ·99795 | ·02 | ·99874 | ·17 | ·99924 |
| ·43 | ·99245 | ·58 | ·99506 | ·73 | ·99683 | ·88 | ·99801 | ·03 | ·99878 | ·18 | ·99926 |
| ·44 | ·99266 | ·59 | ·99520 | ·74 | ·99693 | ·89 | ·99807 | ·04 | ·99882 | ·19 | ·99929 |
| 2·45 | 0·99286 | 2·60 | 0·99534 | 2·75 | 0·99702 | 2·90 | 0·99813 | 3·05 | 0·99886 | 3·20 | 0·99931 |
| ·46 | ·99305 | ·61 | ·99547 | ·76 | ·99711 | ·91 | ·99819 | ·06 | ·99889 | ·21 | ·99934 |
| ·47 | ·99324 | ·62 | ·99560 | ·77 | ·99720 | ·92 | ·99825 | ·07 | ·99893 | ·22 | ·99936 |
| ·48 | ·99343 | ·63 | ·99573 | ·78 | ·99728 | ·93 | ·99831 | ·08 | ·99896 | ·23 | ·99938 |
| ·49 | ·99361 | ·64 | ·99585 | ·79 | ·99736 | ·94 | ·99836 | ·09 | ·99900 | ·24 | ·99940 |
| 2·50 | 0·99379 | 2·65 | 0·99598 | 2·80 | 0·99744 | 2·95 | 0·99841 | 3·10 | 0·99903 | 3·25 | 0·99942 |
| ·51 | ·99396 | ·66 | ·99609 | ·81 | ·99752 | ·96 | ·99846 | ·11 | ·99906 | ·26 | ·99944 |
| ·52 | ·99413 | ·67 | ·99621 | ·82 | ·99760 | ·97 | ·99851 | ·12 | ·99910 | ·27 | ·99946 |
| ·53 | ·99430 | ·68 | ·99632 | ·83 | ·99767 | ·98 | ·99856 | ·13 | ·99913 | ·28 | ·99948 |
| ·54 | ·99446 | ·69 | ·99643 | ·84 | ·99774 | ·99 | ·99861 | ·14 | ·99916 | ·29 | ·99950 |
| 2·55 | 0·99461 | 2·70 | 0·99653 | 2·85 | 0·99781 | 3·00 | 0·99865 | 3·15 | 0·99918 | 3·30 | 0·99952 |

The critical table below gives on the left the range of values of $x$ for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | 0·9994 | | 0·99990 | | 0·99995 |
| 3·075 | | 3·263 | | 3·731 | | 3·916 | |
| | 0·9990 | | 0·9995 | | 0·99991 | | 0·99996 |
| 3·105 | | 3·320 | | 3·759 | | 3·976 | |
| | 0·9991 | | 0·9996 | | 0·99992 | | 0·99997 |
| 3·138 | | 3·389 | | 3·791 | | 4·055 | |
| | 0·9992 | | 0·9997 | | 0·99993 | | 0·99998 |
| 3·174 | | 3·480 | | 3·826 | | 4·173 | |
| | 0·9993 | | 0·9998 | | 0·99994 | | 0·99999 |
| 3·215 | | 3·615 | | 3·867 | | 4·417 | |
| | 0·9994 | | 0·9999 | | 0·99995 | | 1·00000 |

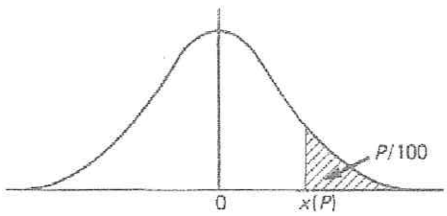
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

# TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If $X$ is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower $P$ per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



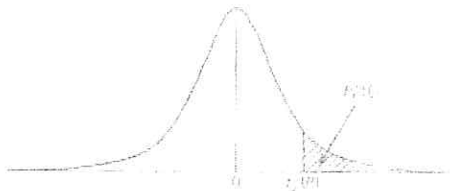

| $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 0·0000 | 5·0 | 1·6449 | 3·0 | 1·8808 | 2·0 | 2·0537 | 1·0 | 2·3263 | 0·10 | 3·0902 |
| 45 | 0·1257 | 4·8 | 1·6646 | 2·9 | 1·8957 | 1·9 | 2·0749 | 0·9 | 2·3656 | 0·09 | 3·1214 |
| 40 | 0·2533 | 4·6 | 1·6849 | 2·8 | 1·9110 | 1·8 | 2·0969 | 0·8 | 2·4089 | 0·08 | 3·1559 |
| 35 | 0·3853 | 4·4 | 1·7060 | 2·7 | 1·9268 | 1·7 | 2·1201 | 0·7 | 2·4573 | 0·07 | 3·1947 |
| 30 | 0·5244 | 4·2 | 1·7279 | 2·6 | 1·9431 | 1·6 | 2·1444 | 0·6 | 2·5121 | 0·06 | 3·2389 |
| 25 | 0·6745 | 4·0 | 1·7507 | 2·5 | 1·9600 | 1·5 | 2·1701 | 0·5 | 2·5758 | 0·05 | 3·2905 |
| 20 | 0·8416 | 3·8 | 1·7744 | 2·4 | 1·9774 | 1·4 | 2·1973 | 0·4 | 2·6521 | 0·01 | 3·7190 |
| 15 | 1·0364 | 3·6 | 1·7991 | 2·3 | 1·9954 | 1·3 | 2·2262 | 0·3 | 2·7478 | 0·005 | 3·8906 |
| 10 | 1·2816 | 3·4 | 1·8250 | 2·2 | 2·0141 | 1·2 | 2·2571 | 0·2 | 2·8782 | 0·001 | 4·2649 |
| 5 | 1·6449 | 3·2 | 1·8522 | 2·1 | 2·0335 | 1·1 | 2·2904 | 0·1 | 3·0902 | 0·0005 | 4·4172 |

# TABLE 10. PERCENTAGE POINTS OF THE $t$-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1 + t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let $X_1$ and $X_2$ be independent random variables having a normal distribution with zero mean and unit variance and a $\chi^2$-distribution with $\nu$ degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's $t$-distribution with $\nu$ degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of $t$ as $\nu$ tends to infinity is the normal distribution with zero mean and unit variance. When $\nu$ is large interpolation in $\nu$ should be harmonic.

| $P$ | 40 | 30 | 25 | 20 | 15 | 10 | 5 | 2.5 | 1 | 0.5 | 0.1 | 0.05 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu = 1$ | 0.3249 | 0.7265 | 1.0000 | 1.3764 | 1.963 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 318.3 | 636.6 |
| 2 | 0.2887 | 0.6172 | 0.8165 | 1.0607 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.33 | 31.60 |
| 3 | 0.2767 | 0.5844 | 0.7649 | 0.9785 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.21 | 12.92 |
| 4 | 0.2707 | 0.5686 | 0.7407 | 0.9410 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | 0.2672 | 0.5594 | 0.7267 | 0.9195 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | 0.2648 | 0.5534 | 0.7176 | 0.9057 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | 0.2632 | 0.5491 | 0.7111 | 0.8960 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| 8 | 0.2619 | 0.5459 | 0.7064 | 0.8889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | 0.2610 | 0.5435 | 0.7027 | 0.8834 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| 10 | 0.2602 | 0.5415 | 0.6998 | 0.8791 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | 0.2596 | 0.5399 | 0.6974 | 0.8755 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | 0.2590 | 0.5386 | 0.6955 | 0.8726 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| 13 | 0.2586 | 0.5375 | 0.6938 | 0.8702 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| 14 | 0.2582 | 0.5366 | 0.6924 | 0.8681 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| 15 | 0.2579 | 0.5357 | 0.6912 | 0.8662 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | 0.2576 | 0.5350 | 0.6901 | 0.8647 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| 17 | 0.2573 | 0.5344 | 0.6892 | 0.8633 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| 18 | 0.2571 | 0.5338 | 0.6884 | 0.8620 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| 19 | 0.2569 | 0.5333 | 0.6876 | 0.8610 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| 20 | 0.2567 | 0.5329 | 0.6870 | 0.8600 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| 21 | 0.2566 | 0.5325 | 0.6864 | 0.8591 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| 22 | 0.2564 | 0.5321 | 0.6858 | 0.8583 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| 23 | 0.2563 | 0.5317 | 0.6853 | 0.8575 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| 24 | 0.2562 | 0.5314 | 0.6848 | 0.8569 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| 25 | 0.2561 | 0.5312 | 0.6844 | 0.8562 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| 26 | 0.2560 | 0.5309 | 0.6840 | 0.8557 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| 27 | 0.2559 | 0.5306 | 0.6837 | 0.8551 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| 28 | 0.2558 | 0.5304 | 0.6834 | 0.8546 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| 29 | 0.2557 | 0.5302 | 0.6830 | 0.8542 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| 30 | 0.2556 | 0.5300 | 0.6828 | 0.8538 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| 32 | 0.2555 | 0.5297 | 0.6822 | 0.8530 | 1.054 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| 34 | 0.2553 | 0.5294 | 0.6818 | 0.8523 | 1.052 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| 36 | 0.2552 | 0.5291 | 0.6814 | 0.8517 | 1.052 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| 38 | 0.2551 | 0.5288 | 0.6810 | 0.8512 | 1.051 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| 40 | 0.2550 | 0.5286 | 0.6807 | 0.8507 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 50 | 0.2547 | 0.5278 | 0.6794 | 0.8489 | 1.047 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| 60 | 0.2545 | 0.5272 | 0.6786 | 0.8477 | 1.045 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 120 | 0.2539 | 0.5258 | 0.6765 | 0.8446 | 1.041 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| $\infty$ | 0.2533 | 0.5244 | 0.6745 | 0.8416 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |